Let $P(x)$ be the predicate "$\frac{30}{x}$ is an integer". **SCORE: _____ / 6 PTS**

Let $Q(x)$ be the predicate "x is odd".

Let $D = \{1, 3, 4, 9, 15\}$ be the domain of both predicates.

[a] Find the truth set of $P(x)$. **You do NOT need to justify your answer.**

$\{1, 3, 15\}$ 2

[b] Find the truth set of $Q(x)$. **You do NOT need to justify your answer.**

$\{1, 3, 9, 15\}$ 2

[c] Is the statement $P(x) \Rightarrow Q(x)$ true or false ? **Explain very briefly.**

2

YES. TRUTH SET OF $P(x) \subseteq$ TRUTH SET OF $Q(x)$

Write the following statement **informally**. **Avoid ambiguous language.** **SCORE: _____ / 3 PTS**

Your answer should NOT use the phrases "for all", "for every", "for each", "for any", "such that", "there exists".

$\exists r \in P : \forall t \in E, \sim V(r, t)$ where P = set of all people, E = set of all countries in Europe,

and $V(r, t)$ = "r has visited t"

SOMEONE HAS NEVER VISITED EUROPE

Consider the statement "All integers between 61 and 67 are composite". **SCORE: _____ / 6 PTS**

[a] Write the statement symbolically, using **TWO** predicates. **State clearly the domain and predicates.**

$\frac{1}{2}$ $P(x)$ = "x IS BETWEEN 61 AND 67"

$\frac{1}{2}$ $Q(x)$ = "x IS COMPOSITE"

1 $\forall x \in \mathbb{Z}, P(x) \rightarrow Q(x)$

PREDICATES MUST HAVE A VARIABLE (X) IN THE TEXT

[b] Write the negation of the statement symbolically, using the domain and predicates from [a].

1 $\exists x \in \mathbb{Z} : P(x) \wedge \sim Q(x)$ 1

[c] Write the inverse of the statement symbolically, using the domain and predicates from [a].

1 $\forall x \in \mathbb{Z}, \sim P(x) \rightarrow \sim Q(x)$ 1

CONTINUED ➡

Let P be the set of all people, and let W be the set of all women. **SCORE: _____ / 6 PTS**

[a] Write the following statement symbolically, using **TWO** quantifiers and **TWO** variables. **State clearly the predicate.**

"Everyone has a mother."

$M(x, y) =$ "y IS THE MOTHER OF x", 1

PREDICATE MUST HAVE 2 VARIABLES IN TEXT

1 $\forall x \in P, \exists y \in W : M(x, y)$ $\frac{1}{2}$

1

PUNCTUATION (, :) MUST MATCH

[b] Write the negation of the statement symbolically.

1 $\exists x \in P : \forall y \in W, \sim M(x, y)$ $\frac{1}{2}$

1

Consider the argument **SCORE: _____ / 7 PTS**

All perfect numbers are even.
37 is not perfect.
Therefore, 37 is not even.

[a] Write the argument symbolically, using **TWO** predicates. **State clearly the domain and predicates.**

1 $P(x) =$ "x IS PERFECT",

1 $Q(x) =$ "x IS EVEN"

PREDICATES MUST HAVE A VARIABLE (x) IN THE TEXT

$\forall x \in \mathbb{R}, P(x) \rightarrow Q(x)$ 2

$\sim P(37)$ $\frac{1}{2}$

$\therefore \sim Q(37)$ $\frac{1}{2}$

[b] Is the argument an example of Universal Instantiation (UI), Universal Modus Ponens (UMP), Universal Modus Tollens (UMT), Universal Transitivity (UT), Converse Error (CE), Inverse Error (IE) or none of the above (NA) ?

IE 2

Let $A = \{-2, 1, 3\}$ and $B = \{-2, 0, 2\}$. **SCORE: _____ / 7 PTS**

Let $P(x, y)$ be the predicate "$x^2 + y$ is a multiple of 3" with domain $A \times B$ (ie. $x \in A$ and $y \in B$).

Determine if the statement "$\exists y \in B : \forall x \in A, P(x, y)$" is true or false.

Justify your answer as shown in lecture. Use as few examples/counterexamples as you need.

$y = -2$ 2 "$\forall x \in A$, $x^2 - 2$ IS A MULTIPLE OF 3" IS FALSE eg. $x = -2$,

$y = 0$ 2 "$\forall x \in A$, x^2 IS A MULTIPLE OF 3" IS FALSE eg. $x = -2$,

$y = 2$ 2 "$\forall x \in A$, $x^2 + 2$ IS A MULTIPLE OF 3" IS FALSE eg. $x = 3$,

THE STATEMENT IS FALSE, 1